KEY

MATH 10-Elementary Statistics

Practice Exam III

TRUE/FALSE

F 1. The mean of a sample mean is equal to the standard deviation divided by the sample size.

T 2. If the standard deviation of a population is 35 and n =49, the standard error of the mean is 5. $\frac{\sigma}{\sqrt{n}} = 35/\sqrt{49} = 5$

F 3. If the variance of a large population is 100 and n =64, the standard error of the mean is 12.5. $\sigma/\sqrt{n} = 10/\sqrt{64} = 1.25$

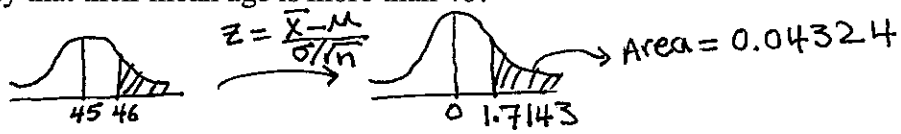
T 4. If N = 600 and n = 150, the standard error of the mean will have to be computed using the finite population correction factor. check $n > .05N$; $150 > .05(600)$, $150 > 30$ yes

T 5. The central limit theorem states that the means of the sample means will be the same as the population mean.

MULTIPLE CHOICES

6. The mean of a population is 400 and the standard deviation is 50. A sample of size 225 is selected. What is the standard error of the mean?
   a) 0.22
   (b) 3.33 $\frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{225}} = 3.33$
   c) 1.78
   d) 8

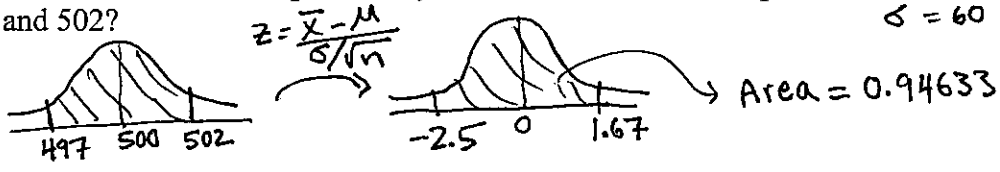
7. The average of farm workers in the United States was thought to be 45 with a standard deviation of 7 years. If 144 farm workers are selected at random, what is the probability that their mean age is more than 46?
   a) 0.4564
   b) 0.4554
   c) 0.0446
   (d) 0.0436



8. If N = 20,000 and n = 1,800, what is the finite population correction factor?
   (a) 0.95
   b) 0.91
   c) 0.83
   d) 0.98

$\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{20{,}000-1{,}800}{20{,}000-1}} = 0.9539$

9. A standard test has a mean of 500 and a standard deviation of 60. If this test is given to 2,500 students, what is the probability that the mean of this sample is between 497 and 502? $\mu = 500$, $n = 2500$, $\sigma = 60$
   (a) 0.9463
   b) 0.8643
   c) 0.9054
   d) 0.7509



10. What is the best point estimate of the mean of the population?
    a) mode of the sample mean
    (b) mean of the sample
    c) median of the sample

d) standard deviation the sample

$\alpha = 1 - .98 = .02$, $\alpha/2 = .01$

11. A sample of the ACT scores of 50 students in Basic Math revealed a mean score of 10 with a standard deviation of 3. What is the 98% confidence interval of the mean for these students?
    a) 9.13-10.87
    b) 9.86-10.14
    c) 9.01-10.99
    d) 9.75-10.75

$n = 50$, $\bar{x} = 10$, $\sigma = 3$, $z_{.01} = 2.33$

$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$10 - \frac{2.33(3)}{\sqrt{50}} < \mu < 10 + \frac{2.33(3)}{\sqrt{50}}$

$9.01 < \mu < 10.99$

12. A political candidate wants to estimate his chances of winning the coming election for mayor to within 2% with 99% confidence. He believes that 54% of the people will vote for him. What is the minimum sample size necessary?
    a) 4,138
    b) 3,372
    c) 4,134
    d) 4,160

$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.54)(1-.54)\left(\frac{2.58}{.02}\right)^2 = \boxed{4134}$

$z_{\alpha/2} = z_{.005} = 2.58$, $\hat{p} = 54\% = .54$, $\hat{q} = 1 - \hat{p} = .46$, $E = 2\% = .02$

13. Ten dieters kept track of how many glasses of water they drank each day. Their data were 4, 6, 7, 8, 5, 7, 4, 8, 3, 5. What is the 95% confidence interval of the true mean?
    a) 4.436-6.964
    b) 4.310-5.105
    c) 3.120-8.051
    d) 5.458-7.235

$n = 10$, $d.f = 9$, $\bar{x} = 5.7$, $s = 1.77$, $t_{\alpha/2} = t_{\frac{.05}{2}} = t_{.025} = 2.26$

$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$

$5.7 - 2.26\frac{1.77}{\sqrt{10}} < \mu < 5.7 + 2.26\frac{1.77}{\sqrt{10}}$

$4.4350 < \mu < 6.964$

14. A recent survey at Sun Valley found that 80 out 120 vacationers chose the resort because of its good ski slopes. What is the 99% confident interval for the proportion of all vacationers who choose Sun Valley for its ski slopes?
    a) 0.555-0.878
    b) 0.555-0.778
    c) 0.456-0.875
    d) 0.321-0.543

$\hat{p} = \frac{80}{120} = \frac{2}{3}$, $\hat{q} = 1 - \frac{2}{3} = \frac{1}{3}$, $n = 120$

$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\frac{2}{3} - 2.58\sqrt{\frac{2/9}{120}} < p < \frac{2}{3} + 2.58\sqrt{\frac{2/9}{120}}$

$0.5556 < p < 0.778$

SHOW WORK

15. The age of six members of a board of directors of a nonprofit organization are shown below.

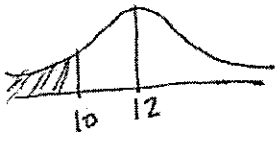
    Member: A, B, C, D, E, F
    Ages: 32, 52, 43, 64, 41, 50

Consider these board members to be a population of interest. The mean age for the population is 47. Construct a table that shows all of the possible sample size of two. For each of the possible samples, list the people in the sample, their ages, and the sample mean. Use your table to find the probability that, for a random sample of size two, the sample mean will equal the population mean.

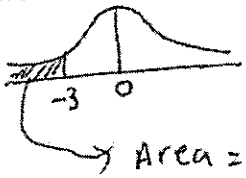
sample	Ages	$\bar{x}$
A, B	32, 52	42
A, C	32, 43	37.5
A, D	32, 64	48
A, E	32, 41	36.5
A, F	32, 50	41
B, C	52, 43	47.5
B, D	52, 64	58
B, E	52, 41	46.5
B, F	52, 50	51
C, D	43, 64	53.5
C, E	43, 41	42
C, F	43, 50	46.5
D, E	64, 41	52.5
D, F	64, 50	57
E, F	41, 50	45.5

Prob. that sample means equals population mean $= 0$

16. The average number of commercials on TV during a half-hour program was 12 with a standard deviation of 4. If 36 programs are selected at random, what is the probability that the average number of commercials is less than 10?



$\rightarrow z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

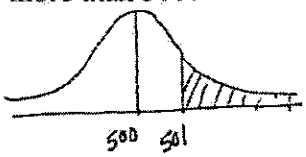


$z = \frac{10-12}{4/\sqrt{36}} = \frac{-2(6)}{4} = -3$

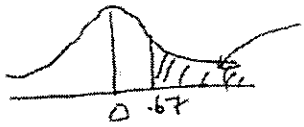
Area = 0.00135

17. A standard test has a mean of 500 and a standard deviation of 60. If this test is given to 1, 600 students, what is the probability that the mean of this sample is more than 501?

$\mu = 500$
$\sigma = 60$
$n = 1600$



$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$



Area = .2514

18. What is the critical value for a 96% confidence interval, using the standard normal distribution?

$z_{\alpha/2} = ?$ $\quad \alpha = 1 - .96 = .04$

$z_{\frac{.04}{2}} = z_{.02} = \boxed{2.054}$

19. What is the critical value for a 90% confidence interval, using a t-distribution with n = 28?

$t_{\alpha/2} = ?$ $\quad \alpha = 1 - .90 = .10$

$df = 28 - 1 = 27$

$t_{\frac{.10}{2}} = t_{.05} = \boxed{1.703}$

20. A class of 15 second-grade students was given a test to see how many simple addition problems they could do in 3 minutes. The results were as follows: 15, 25, 32, 18, 23, 35, 16, 23, 30, 19, 27, 33, 17, 29, 23. What is the 90% confidence interval of the true mean?

$n = 15$
$\bar{x} = 24.33$
$s = 6.51$

$\alpha = 1 - .90 = .10$

$t_{\alpha/2} = t_{.05} = 1.7613$

$df = 15 - 1 = 14$

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$24.33 - 1.7613\frac{6.51}{\sqrt{15}} < \mu < 24.33 + 1.7613\frac{6.51}{\sqrt{15}}$$

$$21.4 < \mu < 27.3$$